OFFICE OF SCIENTIFIC RESEARCH & DEVELOPMENT

Routing Slip

MAR 20 1947

TO W.W. - WJ

We are looking forward to seeing you shortly when you can see in more detail what has been going on here than can be conveyed readily in a progress report.

It will be good to see you again.

H.L.H.

FROM: DATE:

6-4303

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING

CAMBRIDGE 39, MASSACHUSETTS

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MIT
Electronic Computation

February 10th, 1947.

Dr. Warren Weaver,
Rockefeller Foundation,
Room 5500,
49 West 49th Street,
New York City, N.Y.

Dear Dr. Weaver:

Attached is a report to the Rockefeller Foundation on the first six-month's work under the grant made to M.I.T. for an investigation of electronic digital computors for the two-year period beginning July 1st, 1946. Because the initial period of such an investigation is devoted in considerable measure to forming conceptions for a generation of new ideas, this report is fairly brief.

I expect that the content of subsequent reports will grow in definiteness and range of content.

Sincerely yours,

Harold L. Hazen.

Harold L. Hazen.

Progress Report I

Rockefeller Electronic Computer Program

R. Taylor
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Progress Report I

Rockefeller Electronic Computer Program

I Scope of This Report

In this first report of progress under the program of research sponsored by the Rockefeller Foundation, it seems appropriate to review the general purposes and objectives of the project as well as to describe the results accomplished in the period July 1 to December 31, 1946.

Active research in electronic digital methods of computation was suspended by the Center of Analysis during the war-time period because of the need for using all available personnel on work of a more pressing character. Some new ideas were thought out during this period, however, and we were able to resume our research, starting September 1, 1945, on a rather limited scale which had about doubled in activity by June, 1946, just prior to the organization of the program with aid from the Foundation.

In the portions of this report which follow, no effort is made to distinguish between developments which took place before and those which took place after July 1, 1946, since such distinctions bear no relation to the technical worth of these developments.

II Long-Range Objectives

Of first consideration in the design and developments of a large-scale calculator, is the obvious question of what work the machine is expected to perform. For the machine which we are considering here and which is to be operated for the benefit of engineers and scientists generally, the answer is that it must be general purpose, or versatile, in two somewhat different respects.

In the first place, the calculator should be equally at home in computing a large number of solutions for a relatively simple problem and in computing a single solution for a large and complex problem. Actually, if the calculator is to be designed for unattended, automatic operation, no essential difference exists

between the two modes of operation. The point is raised here for the sake of completeness of discussion and because it may influence the design of certain intermediate, experimental apparatus to be described later.

In the second place, the calculator needs mathematical versatility in order to cope with numerical calculations arising from a wide variety of problems. Such problems may be classified, for convenience, into five principal groups which involve, primarily: arithmetic operations, the solutions to algebraic equations, total differential equations, partial differential equations, and integral equations. From a mathematical point of view, this scheme of classification is only moderately significant for two reasons: a), problems originally classified in one particular way may be reclassified if their defining equations are transformed in a suitable manner (not necessarily with benefit to the computer, however), and, b), large problems may be sufficiently complex as to require classification under more than one heading if all phases of the work are to be handled in one operation by the calculator.

From the point of view of machine design and development, however, the classification serves to focus attention on the fact that a machine that is general purpose in the mathematical sense is not necessarily general purpose in the sense that identical, general methods of computation prevail throughout. We have reason to believe that suitable interconnection of highly specialized subcomputing units to constitute "one" calculator may possess very real advantages in comparison with a calculator that must rely entirely on repetitive arithmetic programming for every type of calculation.

In the long run, what we want to obtain is a machine that will accept a problem statement in a form resembling normal mathematical language as closely as possible and that will render solutions in the form most easily and conveniently utilized. We visualize the new calculator as a machine designed not only for

the mass-production of routine problem solutions, but also for use by the mathematician who is conducting a program of research in the field of applied mathematics. A calculator used in this latter fashion becomes the laboratory instrument for applied mathematics, making possible the establishment of experimental laboratories of mathematics whose functions would be to provide the same opportunities for experimentation in mathematics as are enjoyed in other fields of science.

The determination of the limits pertaining to the size of this calculating machine must depend to a large extent on considerations of over-all economics. An estimate of the minimum size of machine can be made as soon as a decision is reached regarding what specific problems and what amount of production work are to be handled; it is difficult to imagine an upper limit on size imposed by mathematical, or technical, considerations only.

III Program in General

Our current two-year program is one of study and basic research rather than one of development and construction. In order to find out how we may best satisfy the extremely broad, general requirements suggested in the preceding section, we are attacking the problem from two different directions, making as few assumptions and using as few preconceived ideas as possible.

We are studying the mathematics of problems to find out whether, or not, there are methods of solution better suited to machine use than the methods now in force for manual computation; we are not adopting the attitude that, because electronic digital computing speeds are high, we are justified in making use of wasteful and inefficient computing routines. If, as a result of our study, the conclusion is reached that the mechanization of an existing manual method is the best solution, then we feel that we are on much firmer ground than we would have been had we made this assumption a priori.

Along with this mathematical study, we are conducting laboratory research on some of the more fundamental types of

apparatus which we feel will be needed regardless of the ultimate structure of the calculator. Our work along these lines is centered on three items at the moment, high-speed counting and whole-number adding circuits, high-speed storage of numbers for internal memory purposes, and low-speed magnetic storage for external memory purposes. It is felt that units of these types will be needed under any circumstances, and we wish to know, as soon as possible, what quality of operation we can reasonably expect to engineer into them, since their speed and reliability of operation will, in general, greatly influence the construction of other more complex components of the calculator.

IV Results of Mathematical Research to Date

One of the crucial problems associated with digital computation is the control of rounding-off and truncation errors generated by the computing process. Closely allied with this problem, particularly with regard to the solution of total differential equations, isthe necessity of superposing on the original equations the various interpolating and integrating ("going-ahead") formulas needed to carry out the required step-by-step solution. Two highly undesirable features result: the calculator must compute using a large number of figures in intermediate calculations in order to avoid overall loss of accuracy; and the computing routine, or programming, is made enormously more complicated and time-consuming for the operator.

As a step in the direction of avoiding some of these difficulties, two new methods have been developed which show considerable promise of success. One method relates to total differential equations, and the other, to algebraic equations.

A. Digital Electronic Integrator

The idea of the digital integrator has been developed with the purpose of retaining the advantages of simplicity and ease of operation associated with the differential analyzer method of solving total differential equations. It might be well to make the definite statement at this point that no attempt is being made to perpetuate the differential analyzer method for any reason other

than whatever technical, or economic, superiority can clearly be shown for it, nor is an attempt being made to extend the differential analyzer method into fields of computation where other methods are better applied. The field of total differential equations is large: the amount of computation associated with it is enormous. The machine that can handle this work-load efficiently and economically needs no further justification for existence. Our plans envisage the design of equipment which will be efficient as an independent unit in the handling of total differential equations, but which will also be capable of functioning (through direct interconnection) in close collaboration with other calculating equipment when such operation is necessary, or desirable. Our thought here is the use of characteristic equations in obtaining the solutions of partial differential equations.

No good terminology has been developed to describe the computation process utilizing the digital integrator. "Unit-increment step-wise computation" for the method and "digital differential analyzer" for the machine are probably the most accurate of the abbreviated descriptions.

Operation of the digital differential analyzer would resemble closely the operation of a mechanical differential analyzer with mechanical inertia effects and integrator slippage removed. Such features as problem set-up, the calculation of scale factors, etc., would be essentially the same in both machines. If one visualizes the rotation of parts in the mechanical analyzer as consisting of very small steps, rather than continuous motion, then a rough, working analogy to the digital machine is obtained.

Computation using the digital analyzer is really step-by-step computation, but exhibiting two fundamental dissimilarities to ordinary step-by-step procedures. First, changes in value of each variable proceed by unit amounts only, that is, the incremental values of all variables corresponding to one step of computaion can only be plus, or minus, one in the right-hand digit of the number representing the value of the variable. Secondly, even

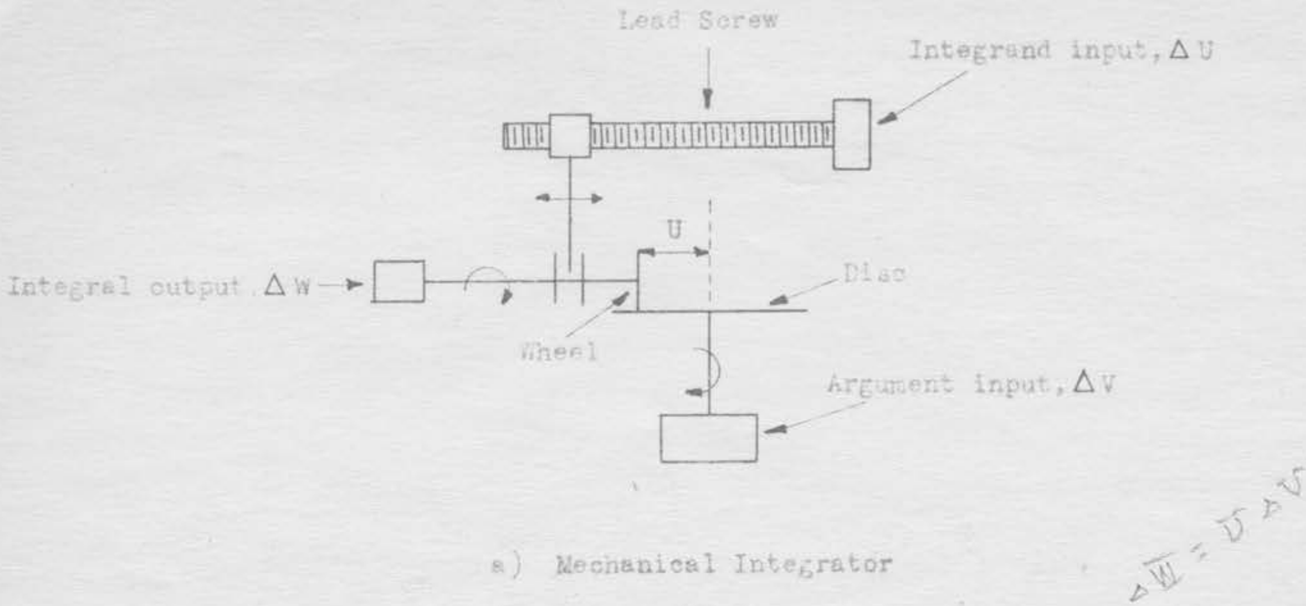
difference real here?
only scale?
impossible to trigger indep. variable twice
before feeding in?

though the interval representing the change in value of the independent variable remains constant, the interval between steps for each dependent variable will, in general, change (with respect to such regular intervals of the independent variable.) This second statement is a direct consequence of the first since each "step" of computation is referred only to the particular variable under consideration and is defined as a change of a constant amount in the value of that variable. The frequency with which steps occur in the course of (computation of a particular variable) at a given time will be approximately proportional to the absolute magnitude of the first derivative of that variable with respect to the individual requirements of each variable at each instant throughout the computation.

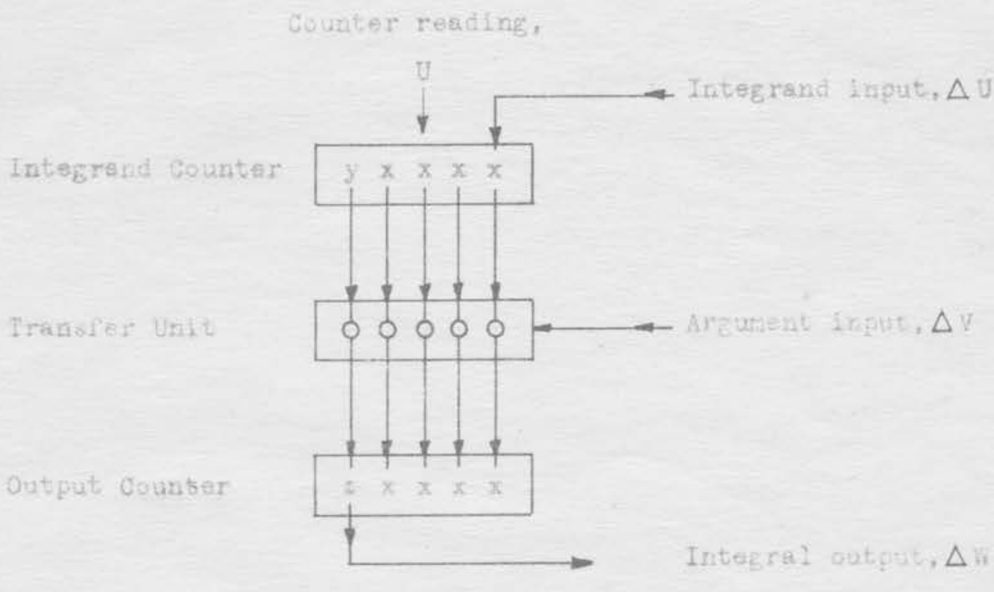
The electronic digital integrator is a simple, straightforward device from the point of view of underlying principle. It consists of the following prinicpal parts, with indication of their correspondence to parts of the mechanical integrator. (See, also, the diagrams on the next page).

Integrand counter, U,	corresponding to the	leadscrew,
Output counter, W,	" " "	integrating wheel,
Transfer unit, V,	" " "	disc and disc input servomechanism.

Referring to figure 1, the basic operation of the digital integrator may be explained as follows. The number of digit positions, marked "x" in the diagram, provided in the integrand and output counters (auxiliary devices excluded) will be the same and will be as great as needed to achieve the accuracy required in the computation. The integrand counter will receive plus, or minus, impulses from some source causing the counter to add, or subtract, one for each impulse in its right-hand digit position in conformity with incremental changes in the value of the variable it represents.



a) Mechanical Integrator



b) Digital Integrator

Figure 1. Schematic diagrams for the two types of integrators.

The receipt of a positive, or negative, impulse by the transfer unit will cause the reading in the integrand counter to be added to, or subtracted from, the reading in the output counter. Whenever the amount accumulated in the output counter is sufficient to cause carry-over from its extreme left-hand "x" position, then the variable represented by this counter has changed by one unit-increment and the integrator will accordingly transmit a positive, or negative, impulse to an appropriate receiving counter located elsewhere in the machine. This receiving counter will receive the impulses from the integrator in its right-hand digit position and will add them to, or subtract them from, whatever reading may already be present.

The scale conversion factor of the integrator may be explained by reference to the digit positions shown as "y" and "z" in figure 1. If the integrand counter contains a "1" in position "y" and zeros elsewhere, then a "1" will be transferred to position "z" of the output counter each time an increment ΔV is received by the transfer unit. The "z" position will transmit an output impulse for every "1" received from the "y" position in the same way that it transmits an output impulse for every carry impulse received from the "x" position adjacent to it. Under these conditions, the integrator transmits a ΔW impulse for every ΔV received; the reading of the integrand counter, therefore, corresponds to unity since $\Delta W = U (\Delta V)$ and $\Delta W = \Delta V$. Digit positions corresponding to "y" and "z" may, or may not, be present as physical units, depending on final integrator design.

The reading of the output counter of an integrator, following the transmission of a carry impulse from its left-hand digit position, represents a fraction of the integral whose magnitude is less than the smallest incremental value of the integral being considered in the computation. To illustrate this point, let us consider the situation existing in a mechanical integrator where the servomechanism connected to the integrating wheel transmits impulses to an external counter only upon the completion of whole revolutions by the wheel. The integrating wheel represents a means whereby

to number of places [illegible] in Int. Counter

exceedingly small increments in the value of the integral may be accumulated until their sum exceeds a previously-determined unit-incremental amount which is the smallest measured fraction of the variable to be considered in the computation.

The writer will report later in more detail on questions of probable error to be encountered and the speed of operation to be expected in this method of computation. There are two definite advantages to be gained in the digital analyzer over the usual step-by-step methods which might be worth mentioning at this time. Since each variable assumes every intermediate value in progressing from its initial value to its final value, no interpolating mechanism need be provided to compute these intermediate values. Secondly, since connections among the various components of the digital analyzer ? remain fixed for the duration of a particular solution, high-speed (electronic speed) switching need not be provided for such inter-connections. assuming they are available

The successful development of the digital analyzer depends primarily on obtaining reliable, high-speed operation of the digital integrator. Since transfer of figures from the integrand counter to the output counter takes place every time an incremental change occurs in the argument of integration, this transfer must take place at as high a speed as possible because it determines the maximum operating speed of the calculator. Laboratory research is being pushed in this direction, but it is too early to state what computing speeds we may be able to obtain.

B. Linear Analyzer

In searching for a method of solving algebraic equations which would be better suited to machine characteristics than any of the classical methods commonly employed today, a process was uncovered which seems to be highly amenable to machine techniques.

This process is essentially one of replacing the set of algebraic equations by a set of total differential equations whose solutions converge in the limit to values of the unknowns that satisfy the original algebraic equations.

Let us first consider the general system of linear algebraic equations:

$$
(1) \quad
\begin{array}{l}
a_{11}\, x_1 + a_{12}\, x_2 + a_{13}\, x_3 + \dots a_{1n}\, x_n = c_1 \\
a_{21}\, x_1 + a_{22}\, x_2 + a_{23}\, x_3 + \dots a_{2n}\, x_n = c_2 \\
- \; - \; - \; - \; - \; - \; - \; - \; - \; - \; - \; - \; - \; - \\
a_{n1}\, x_1 + a_{n2}\, x_2 + a_{n3}\, x_3 + \dots a_{nn}\, x_n = c_n
\end{array}
$$

where the x's are the unknowns and the a's and c's are known constants. These equations may be abbreviated to matrix algebra form and written as

$$AX = C,$$

where A represents the square matrix of coefficients, a_{ij}; X represents the column matrix of the unknowns, x_i; C represents the column matrix of the constant terms, c_i; and the subscripts, i and j, may take on all integral values from 1 to n, inclusive.

As the first step, using the first equation of set (1) as an example, let us introduce a new quantity, r, and re-arrange the equation in the following manner:

$$(2) \quad a_{11}\, x_1 + a_{12}\, x_2 + a_{13}\, x_3 + \dots a_{1n}\, x_n - c_1 = r_1$$

For the full set of equations in matrix form,

$$AX - C = R.$$

Now, let us introduce the parameter, or variable of computation, t, and let the unknowns, x_i, and the new quantities, r_i, be functions of t.

The variables, r_i, termed the residuals, may now be defined as functions whose instantaneous values indicate the amounts by which the equations (1) fail to be satisfied when a specified set of values of $x_i(t)$ are substituted in them. If the value of the determinant of the matrix, A, is different from zero, then the values of $x_i(t)$ that cause all of the r_i's to be equal to zero at the same time are the values of the unknowns we seek.

The values of the x_i's required above may be calculated with the help of the following total differential equations:

$$(3) \quad \dot{x}_1 = -(a_{11}\, r_1 + a_{21}\, r_2 + a_{31}\, r_3 + \dots a_{n1}\, r_n)$$
$$\dot{x}_2 = -(a_{12}\, r_1 + a_{22}\, r_2 + a_{32}\, r_3 + \dots a_{n2}\, r_n), \text{ etc.}$$

In matrix form, these equations may be expressed,

$$\dot{X} = -A'R,$$

where A' is the transpose of the matrix, A.

By the application of the principles of matrix algebra, it can be shown that the solutions of equations (3) converge to solutions of equations (1) for any matrix, A, whose determinant is not equal to zero. Proof that this is so rests upon the demonstration that the solutions of equations (3) are in the form,

$$(4) \quad X = [1 - \exp(-A'At)]\, A^{-1}C, \text{ if } X_0 = 0,$$
$$\text{or, } X = A^{-1}C + [\exp(-A'At)]\,(X_0 - A^{-1}C), \text{ if } X_0 \neq 0,$$

where A^{-1} is the inverse of the matrix, A.

The equation, $X = A^{-1}C$, is a restatement of the equation, $AX = C$, giving X explicitly in terms of the known quantities. If the expression, $\exp(-A'At)$, can be shown to vanish as t approaches infinity, then equations (4) converge, and the process yields the required results. The expression in question does vanish as t approaches infinity because it is composed entirely of terms having the form $b_i \exp(-\lambda_j t)$, where the b's and λ's are constants, the λ's being the latent roots of the matrix, $A'A$. The matrix, $A'A$, is always positive-definite, so that the λ's are always positive numbers. Each exponential term, $b \exp(-\lambda t)$, will approach zero, therefore, as t approaches infinity.

In the foregoing discussion, the coefficients, a_{ij}, have been assumed to be real numbers. If these coefficients are complex, then the method will converge if one uses $\dot{X} = -A^*R$ in place of $\dot{X} = -A'R$, where A^* is the conjugate transposed matrix of A.

It can be shown that this method of solution of algebraic equations is a process of minimization. If there are more equations to be satisfied than there are unknowns, so that the matrix, A, is rectangular rather than square, then the process will yield values

for the x's which make the sum of squares of the r's a minimum.

It can also be shown that this method is applicable to sets of non-linear algebraic equations. Under these circumstances, the transposed matrix, A', is replaced by the Jacobian, J. (The transposed matrix, A', is, of course, the Jacobian of the matrix, AX). The Jacobian for non-linear sets will contain elements which are variables, whereas all of the elements of the matrix, A', are constants. For this reason, application of this process to non-linear equations is, in general, quite complicated, particularly since consideration must be given to the matter of the determination of the values of multiple roots.

Only one special class of non-linear algebraic equations seemed to us of sufficient immediate importance to justify the expenditure of much effort at this time; this is the class of _non-linear_ equations which results where the attempt is made to solve for the latent roots of the matrix, A, associated with a set of _linear_ equations. We have made some head-way along these lines, but I would prefer to report on this subject at a later date when more definite information is available.

The method of computation outlined in the preceding paragraphs, we found after we had worked it all out, is the limit which would be reached by two well-known iterative procedures as the number of iterative steps increases without limit and as the size of the step, or interval, decreases without limit. These two iterative procedures are the Hotelling method applied to linear equations and the method of steepest descent applied to non-linear equations.

Our principal concern at the present time is the application of this process to the solution of linear algebraic equations. The ability to solve large sets of linear equations is useful not only when such sets of equations arise "naturally" in the course of scientific and engineering studies, but also when they arise through the transformation of certain classes of partial differential equations to the form of difference equations. A really satisfactory method of solving the algebraic equations may well prove to be a

valuable method of treatment for a large class of partials.

The procedure, as outlined for linear algebraic equations, has been tried out using the differential analyzer, with promising results. Because of the large number of gear-boxes and adder-units required, it does not seem at all feasible to attempt to use a differential-analyzer type of machine for this purpose when the *how many?* number of equations involved may be in the order of fifty to one hundred. We have in mind the design of an electric-network type of machine which may yield solutions quickly to an order of accuracy of one percent. By employing electronic-digital methods in substituting this first approximation back into the original equations to obtain new constant terms for the computation of further approximations in accordance with standard procedures, high over-all speed can be attained with arbitrary accuracy in the final results using only a minimum of equipment.

Experimental apparatus operating on these principles has not yet been built. The Research Laboratory for Electronics has agreed to cooperate with us in this research, however, and we expect that laboratory work will be under way in a very short time. ?

V Continuing Program for Mathematical Research

Our current program of mathematical research is centered on the development of methods of computation that are well-suited to the job of obtaining numerical solutions to partial differential equations and to integral equations, with principal attention being paid to the former at the instant.

It appears at this stage that it may be entirely appropriate to use more than one machine method, depending on whether the partial differential equations are linear (including quasi-linear) or non-linear, elliptic or hyperbolic.

We are examining methods involving the use of difference equations, as well as those using characteristic equations, but our study has not yet progressed to the point where a definite conclusion may be drawn.

VI Laboratory Research

Laboratory research leading to the design and development of some of the more fundamental components of the calculator is concentrated along three principal channels: high-speed pulse-counting and whole-number adding circuits which will be the basic elements of other more complex units; electric storage for high-speed, internal memory purposes; and magnetic storage which is planned principally for data-handling and machine control, but which may also be suitable for use as part of the internal memory in certain cases.

Much of the time since July 1 has had to be devoted to the physical organization of the laboratory and to the building of special test equipment, such as pulse sources, oscillators, and power supplies which could not be obtained commercially. Considerable delay has been, and still is being, experienced in obtaining reasonable delivery times for laboratory instruments, parts, and materials, particularly tubes and magnetic materials.

Despite such delays, the laboratory work is now well established, and we are making progress. A formal account of this phase of activity will be included in our July 1947 report.

VII Organization: Personnel and Laboratory Facilities

Our present staff, exclusive of the writer, consists of the following persons: G. B. Thomas, jr, assistant professor of mathematics; R. H. Blythe and F. M. Verzuh, research associates in electrical engineering; E. S. Rich, C. H. R. Campling, and E. W. Sard, research assistants in electrical engineering; J. H. Huska, machinist; and H. Ladd, electrical technician. We expect to increase this staff by the addition of one more research man next June, or July, and by one, or two, more electrical technicians as soon as the requirements of electrical construction demand it.

Quarters for the Electronic Computer project, along with some other Center of Analysis activities, have been provided in Building 20, one of the buildings formerly forming a part of the

Radiation Laboratory. Laboratory and shop facilities are now fully installed, although the last of the machine tools for the shop were received only very recently. The shops for this project are sufficiently well-equipped to enable them to operate on a self-sufficient basis in most respects, although some of the more expensive machine shop items have not been duplicated since the Differential Analyzer shop is available to the project whenever needed.

VIII Financial

Summary of expenditures from the Rockefeller Electronic Computer Account from July 1 to December 31, 1946.

Salaries and wages, ...	$ 7790.35
Laboratory equipment.	. 2686.45
Shop tools and equipment. . . .	. 607.91
Other expenses.	.441.44
Total expenses	. $ 11,526.15

Orders outstanding as of December 31, 1946

Laboratory equipment	$2,162.28

The item of expense for "Salaries and wages" is substantially below our budget estimate for this period because some members of the research staff were not able to join us until late in September and because we have not needed to hire as many shop men over this period as we had anticipated.